

Grey Wolf Optimization Worked by Hand

A free sample from Computational Labs

Four iterations. Five wolves.
Every random draw written out.

Wolf	r_1	r_2	$A = 2a \cdot r_1 - a$	$C = 2 \cdot r_2$
W1	0.3993	0.7174	-0.4028	1.4348
W2	0.2808	0.0827	-0.8767	0.1654
W3	0.9698	0.5639	1.8791	1.1279
W4	0.6443	0.5769	0.5773	1.1537
W5	0.4754	0.1224	-0.0986	0.2448

Table 2. Iteration 1 coefficients. Each wolf draws one (r_1, r_2) ; A and C follow from the Section 4 formulas.

Worked detail for W5. Take its x coordinate, starting from $x = 8.0000$ with $A = -0.0986$, $C = 0.2448$. From alpha ($x_L = -2.0000$): $D = |0.2448 \times -2.0000 - 8.0000| = 8.4896$, so $X_1 = -2.0000 - -0.0986 \times 8.4896 = -1.1633$. The same from beta gives $X_2 = 5.6678$, and from delta $X_3 = -5.0668$. The new x is their average, $(-1.1633 + 5.6678 + -5.0668) / 3 = -0.1874$.

Wolf	Coord	$X_1 (\alpha)$	$X_2 (\beta)$	$X_3 (\delta)$	Average	New
W1	x	1.1697	5.8757	-0.5186	2.1756	2.1756
	y	-0.4993	4.7006	3.1226	2.4413	2.4413
W2	x	2.9701	10.9855	-1.6101	4.1152	4.1152
	y	-1.6446	6.0499	5.1950	3.2001	3.2001
W3	x	-19.3923	2.4431	-31.8696	-16.2729	-10.0000 *
	y	-9.4360	-19.5100	-18.3907	-15.7789	-10.0000 *
W4	x	-2.1775	0.5152	-8.8417	-3.5013	-3.5013
	y	-5.4438	-1.5506	-1.8845	-2.9596	-2.9596
W5	x	-1.1633	5.6678	-5.0668	-0.1874	-0.1874
	y	-4.2880	4.4948	3.5190	1.2419	1.2419

An unretouched page from the by-hand file.

This is File 2 of five. The volume also contains the theory,
a live spreadsheet, commented Python, and a reusable prompt.

Learning Grey Wolf Optimization

A Worked Example, Solved by Hand

File 2 of 5 · The Grey Wolf Optimization (GWO) Volume

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1. What this file does

This file solves one small optimization problem completely by hand using Grey Wolf Optimization. Everything is written out so you can follow along with a calculator and reproduce every number. The same problem is carried further in the spreadsheet (File 3) and coded up in Python (File 4), so once you have followed the hunt here you will recognize it everywhere else in this volume.

Grey Wolf Optimization imitates how a pack of grey wolves hunts. A few leading wolves sense roughly where the prey is, and the rest of the pack repositions around them. In optimization terms, the best three solutions found so far act as guides, and every other candidate solution steps toward a point suggested by those three. As the hunt goes on, the wolves tighten the circle. That is the whole idea, and the arithmetic below shows it happening.

2. The problem and its data

We minimize the two dimensional sphere function

$$f(x, y) = x^2 + y^2$$

over the square region where x and y each lie between **-10 and 10**. This function is a smooth bowl centered on the origin. Its lowest point is at **(0, 0)**, where $f = 0$. Because we already know the answer, we can watch the pack close in on it and judge how well the method is doing. A wolf here is simply a point (x, y) ; its fitness is the value of f at that point, and smaller is better.

The pack has five wolves. We start them at fixed, round positions so the walkthrough has a definite beginning:

Wolf	x	y	$f = x^2 + y^2$	Rank
W1	5.0000	4.0000	41.0000	2 (beta)
W2	-6.0000	3.0000	45.0000	3 (delta)
W3	7.0000	-8.0000	113.0000	5 (omega)
W4	-2.0000	-5.0000	29.0000	1 (alpha)
W5	8.0000	6.0000	100.0000	4 (omega)

Table 1. The initial pack. The three lowest f values become the guides for the first hunt: alpha (W4), beta (W1), and delta (W2).

3. Assumptions that make every step exact

Grey Wolf Optimization uses random numbers, so to make the walkthrough reproducible we fix a few conventions up front. Follow these and you will get exactly the numbers printed here.

- **Encoding.** A wolf is a pair (x, y) of real numbers. Nothing is rounded during the hunt except for display; the tables show four decimals, but the computation carries full precision.
- **Leaders.** At the start of each iteration the pack is ranked by f. The best is alpha, the second best is beta, the third best is delta. Every wolf, including the leaders themselves, then moves.
- **The move.** Each wolf computes a suggested landing point from each of the three leaders and averages them. The formulas are in Section 4.
- **Control parameter a.** a starts at 2 and falls in steps of 0.2 each iteration, so $a = 2 - 0.2(k - 1)$ for iteration k, held at 0 once it reaches 0. Large a early lets wolves roam; small a late pulls them tight around the leaders.
- **Random numbers.** To keep the arithmetic light enough for hand work, we draw one pair of random numbers (r_1, r_2) per wolf per iteration and reuse it across the three leaders and both coordinates. Standard GWO redraws these for every leader and coordinate; our teaching convention fixes them so the pack can be checked by hand. The Python file uses the identical convention, with a switch to the fully standard draw.
- **Seed.** The random pairs come from a fixed generator (NumPy default_rng with seed 18), drawn in order W1, W2, ..., W5 each iteration. They are listed in every iteration below, so you never have to generate them yourself.
- **Boundaries.** If a computed coordinate falls outside [-10, 10] it is clamped back to the nearest boundary. You will see this happen to one wolf below.
- **Tie-breaking.** If two wolves have equal fitness, the one with the smaller index (W1 before W2, and so on) ranks higher.
- **Stopping.** We run four iterations by hand. That is enough to see the pack close in; File 3 carries the same run to fifteen iterations.
- **Best-so-far.** We separately remember the best solution seen at any point. A pack can drift to a slightly worse spot in a given step, so keeping the best-so-far guarantees we never lose our best answer.

4. The formulas used at every step

For a wolf at position X guided by one leader at position X_L , in a single coordinate:

$$A = 2a \cdot r_1 - a \quad C = 2 \cdot r_2$$

$$D = |C \cdot X_L - X| \quad X' = X_L - A \cdot D$$

The wolf does this for each of the three leaders (alpha, beta, delta), producing three suggested coordinates X_1, X_2, X_3 . Its new coordinate is their average:

$$X_{\text{new}} = (X_1 + X_2 + X_3) / 3$$

Both coordinates x and y are updated this way. Because A can exceed 1 in size early on, a wolf can be pushed well away from a leader (exploration); as a shrinks, A shrinks, and wolves settle onto the leaders (exploitation). In our convention a wolf uses the same A and C for all three leaders and both coordinates, so each iteration needs just the one pair (r_1, r_2) per wolf.

5. The hunt, iteration by iteration

Each iteration below ranks the pack, lists the random draws, moves every wolf, and evaluates the result. Watch the best-so-far value fall as the wolves close in on the origin.

5.1 Iteration 1 (a = 2.0000)

The pack is ranked. The guides for this iteration are **alpha = W4** at (-2.0000, -5.0000), **beta = W1** at (5.0000, 4.0000), and **delta = W2** at (-6.0000, 3.0000). With a = 2.0000, each wolf now moves.

Wolf	r_1	r_2	$A = 2a \cdot r_1 - a$	$C = 2 \cdot r_2$
W1	0.3993	0.7174	-0.4028	1.4348
W2	0.2808	0.0827	-0.8767	0.1654
W3	0.9698	0.5639	1.8791	1.1279
W4	0.6443	0.5769	0.5773	1.1537
W5	0.4754	0.1224	-0.0986	0.2448

Table 2. Iteration 1 coefficients. Each wolf draws one (r_1 , r_2); A and C follow from the Section 4 formulas.

Worked detail for W5. Take its x coordinate, starting from $x = 8.0000$ with $A = -0.0986$, $C = 0.2448$. From alpha ($x_L = -2.0000$): $D = |0.2448 \times -2.0000 - 8.0000| = 8.4896$, so $X_1 = -2.0000 - -0.0986 \times 8.4896 = -1.1633$. The same from beta gives $X_2 = 5.6678$, and from delta $X_3 = -5.0668$. The new x is their average, $(-1.1633 + 5.6678 + -5.0668) / 3 = -0.1874$.

Wolf	Coord	$X_1 (\alpha)$	$X_2 (\beta)$	$X_3 (\delta)$	Average	New
W1	x	1.1697	5.8757	-0.5186	2.1756	2.1756
	y	-0.4993	4.7006	3.1226	2.4413	2.4413
W2	x	2.9701	10.9855	-1.6101	4.1152	4.1152
	y	-1.6446	6.0499	5.1950	3.2001	3.2001
W3	x	-19.3923	2.4431	-31.8696	-16.2729	-10.0000 *
	y	-9.4360	-19.5100	-18.3907	-15.7789	-10.0000 *
W4	x	-2.1775	0.5152	-8.8417	-3.5013	-3.5013
	y	-5.4438	-1.5506	-1.8845	-2.9596	-2.9596
W5	x	-1.1633	5.6678	-5.0668	-0.1874	-0.1874
	y	-4.2880	4.4948	3.5190	1.2419	1.2419

Table 3. Iteration 1 position updates. Each coordinate is the average of the three leader suggestions. A star (*) marks a value clamped to the [-10, 10] boundary.

Wolf	new x	new y	$f = x^2 + y^2$	Rank
W1	2.1756	2.4413	10.6932	2
W2	4.1152	3.2001	27.1753	4
W3	-10.0000	-10.0000	200.0000	5
W4	-3.5013	-2.9596	21.0185	3
W5	-0.1874	1.2419	1.5775	1

Table 4. Iteration 1 evaluation and new ranking.

After the move, the best wolf is **W5** with $f = 1.5775$ at (-0.1874, 1.2419). This is a new best-so-far.

5.2 Iteration 2 (a = 1.8000)

The pack is ranked. The guides for this iteration are **alpha = W5** at (-0.1874, 1.2419), **beta = W1** at (2.1756, 2.4413), and **delta = W4** at (-3.5013, -2.9596). With $a = 1.8000$, each wolf now moves.

Wolf	r_1	r_2	$A = 2a \cdot r_1 - a$	$C = 2 \cdot r_2$
W1	0.3136	0.7362	-0.6711	1.4724
W2	0.9074	0.8886	1.4666	1.7772
W3	0.9483	0.0254	1.6140	0.0508
W4	0.7380	0.6726	0.8569	1.3452
W5	0.6252	0.6376	0.4507	1.2752

Table 5. Iteration 2 coefficients. Each wolf draws one (r_1, r_2); A and C follow from the Section 4 formulas.

Worked detail for W5. Take its x coordinate, starting from $x = -0.1874$ with $A = 0.4507$, $C = 1.2752$. From alpha ($x_L = -0.1874$): $D = |1.2752 \times -0.1874 - -0.1874| = 0.0516$, so $X_1 = -0.1874 - 0.4507 \times 0.0516 = -0.2107$. The same from beta gives $X_2 = 0.8408$, and from delta $X_3 = -5.4291$. The new x is their average, $(-0.2107 + 0.8408 + -5.4291) / 3 = -1.5997$.

Wolf	Coord	$X_1 (\alpha)$	$X_2 (\beta)$	$X_3 (\delta)$	Average	New
W1	x	1.4577	2.8653	1.4182	1.9137	1.9137
	y	1.6531	3.2152	1.6029	2.1571	2.1571
W2	x	-6.7113	1.8108	-18.6626	-7.8543	-7.8543
	y	-0.2143	0.7714	-15.3670	-4.9366	-4.9366
W3	x	-16.3116	-14.1424	-19.3537	-16.6026	-10.0000 *
	y	-14.9995	-13.8984	-18.8565	-15.9181	-10.0000 *
W4	x	-2.9718	-3.3327	-4.5370	-3.6139	-3.6139
	y	-2.7259	-2.9091	-3.8351	-3.1567	-3.1567
W5	x	-0.2107	0.8408	-5.4291	-1.5997	-1.5997
	y	1.0879	1.5979	-5.2203	-0.8448	-0.8448

Table 6. Iteration 2 position updates. Each coordinate is the average of the three leader suggestions. A star (*) marks a value clamped to the $[-10, 10]$ boundary.

Wolf	new x	new y	$f = x^2 + y^2$	Rank
W1	1.9137	2.1571	8.3153	2
W2	-7.8543	-4.9366	86.0610	4
W3	-10.0000	-10.0000	200.0000	5
W4	-3.6139	-3.1567	23.0249	3
W5	-1.5997	-0.8448	3.2727	1

Table 7. Iteration 2 evaluation and new ranking.

After the move, the best wolf is **W5** with $f = 3.2727$ at $(-1.5997, -0.8448)$. This is worse than an earlier step, so the best-so-far stays at $f = 1.5775$. Keeping that memory is exactly why we track it.

5.3 Iteration 3 (a = 1.6000)

The pack is ranked. The guides for this iteration are **alpha = W5** at (-1.5997, -0.8448), **beta = W1** at (1.9137, 2.1571), and **delta = W4** at (-3.6139, -3.1567). With $a = 1.6000$, each wolf now moves.

Wolf	r_1	r_2	$A = 2a \cdot r_1 - a$	$C = 2 \cdot r_2$
W1	0.1263	0.6262	-1.1959	1.2523
W2	0.7871	0.0074	0.9187	0.0148
W3	0.9210	0.1306	1.3473	0.2613
W4	0.3757	0.5507	-0.3977	1.1014
W5	0.6083	0.5494	0.3464	1.0989

Table 8. Iteration 3 coefficients. Each wolf draws one (r_1 , r_2); A and C follow from the Section 4 formulas.

Worked detail for W4. Take its x coordinate, starting from $x = -3.6139$ with $A = -0.3977$, $C = 1.1014$. From alpha ($x_L = -1.5997$): $D = |1.1014 \times -1.5997 - -3.6139| = 1.8520$, so $X_1 = -1.5997 - -0.3977 \times 1.8520 = -0.8631$. The same from beta gives $X_2 = 4.1895$, and from delta $X_3 = -3.4681$. The new x is their average, $(-0.8631 + 4.1895 + -3.4681) / 3 = -0.0472$.

Wolf	Coord	$X_1 (\alpha)$	$X_2 (\beta)$	$X_3 (\delta)$	Average	New
W1	x	3.0849	2.4912	4.0873	3.2211	3.2211
	y	3.0002	2.8080	4.1508	3.3197	3.3197
W2	x	-8.7937	-5.3280	-10.7804	-8.3007	-8.3007
	y	-5.3686	-2.4075	-7.6490	-5.1417	-5.1417
W3	x	-14.5100	-12.2333	-15.8151	-14.1861	-10.0000 *
	y	-14.0208	-12.0756	-15.5189	-13.8718	-10.0000 *
W4	x	-0.8631	4.1895	-3.4681	-0.0472	-0.0472
	y	0.0406	4.3576	-3.0294	0.4563	0.4563
W5	x	-1.6545	0.6311	-4.4354	-1.8196	-1.8196
	y	-0.8738	1.0433	-4.0657	-1.2987	-1.2987

Table 9. Iteration 3 position updates. Each coordinate is the average of the three leader suggestions. A star (*) marks a value clamped to the [-10, 10] boundary.

Wolf	new x	new y	$f = x^2 + y^2$	Rank
W1	3.2211	3.3197	21.3961	3
W2	-8.3007	-5.1417	95.3391	4
W3	-10.0000	-10.0000	200.0000	5
W4	-0.0472	0.4563	0.2104	1
W5	-1.8196	-1.2987	4.9976	2

Table 10. Iteration 3 evaluation and new ranking.

After the move, the best wolf is **W4** with $f = 0.2104$ at (-0.0472, 0.4563). This is a new best-so-far.

5.4 Iteration 4 (a = 1.4000)

The pack is ranked. The guides for this iteration are **alpha = W4** at (-0.0472, 0.4563), **beta = W5** at (-1.8196, -1.2987), and **delta = W1** at (3.2211, 3.3197). With $a = 1.4000$, each wolf now moves.

Wolf	r_1	r_2	$A = 2a \cdot r_1 - a$	$C = 2 \cdot r_2$
W1	0.3877	0.8803	-0.3144	1.7606
W2	0.3966	0.8735	-0.2895	1.7470
W3	0.3299	0.6257	-0.4763	1.2514
W4	0.8498	0.1198	0.9795	0.2397
W5	0.1709	0.7283	-0.9215	1.4565

Table 11. Iteration 4 coefficients. Each wolf draws one (r_1 , r_2); A and C follow from the Section 4 formulas.

Worked detail for W4. Take its x coordinate, starting from $x = -0.0472$ with $A = 0.9795$, $C = 0.2397$. From alpha ($x_L = -0.0472$): $D = |0.2397 \times -0.0472 - -0.0472| = 0.0359$, so $X_1 = -0.0472 - 0.9795 \times 0.0359 = -0.0824$. The same from beta gives $X_2 = -2.2004$, and from delta $X_3 = 2.4187$. The new x is their average, $(-0.0824 + -2.2004 + 2.4187) / 3 = 0.0453$.

Wolf	Coord	$X_1 (\alpha)$	$X_2 (\beta)$	$X_3 (\delta)$	Average	New
W1	x	0.9916	0.2003	3.9914	1.7278	1.7278
	y	1.2474	0.4638	4.1135	1.9416	1.9416
W2	x	2.3316	-0.3370	7.2527	3.0824	3.0824
	y	2.1753	-0.4672	6.4867	2.7316	2.7316
W3	x	4.6879	1.8591	9.9045	5.4838	5.4838
	y	5.4915	2.6904	10.0618	6.0812	6.0812
W4	x	-0.0824	-2.2004	2.4187	0.0453	0.0453
	y	0.1165	-2.0505	2.9873	0.3511	0.3511
W5	x	1.5661	-1.0541	9.2212	3.2444	3.2444
	y	2.2654	-0.7524	8.9720	3.4950	3.4950

Table 12. Iteration 4 position updates. Each coordinate is the average of the three leader suggestions.

Wolf	new x	new y	$f = x^2 + y^2$	Rank
W1	1.7278	1.9416	6.7548	2
W2	3.0824	2.7316	16.9631	3
W3	5.4838	6.0812	67.0534	5
W4	0.0453	0.3511	0.1253	1
W5	3.2444	3.4950	22.7411	4

Table 13. Iteration 4 evaluation and new ranking.

After the move, the best wolf is **W4** with $f = 0.1253$ at (0.0453, 0.3511). This is a new best-so-far.

6. Convergence criteria

We stopped after four iterations because the pack had clearly found the neighborhood of the optimum: the best-so-far fell from $f = 29$ at the start to $f = 0.1253$ by iteration 4, with the best point at (0.0453, 0.3511), very close to the true optimum at the origin. In practice you would stop when the best-so-far stops improving by more than a small tolerance over several iterations, or when a maximum iteration count is reached. File 3 uses the second rule and runs fifteen iterations.

7. Summary of all iterations

Iteration	a	Pack best f	Best-so-far f	Best-so-far (x, y)
Initial	—	29.0000	29.0000	(-2.0000, -5.0000)
1	2.0000	1.5775	1.5775	(-0.1874, 1.2419)
2	1.8000	3.2727	1.5775	(-0.1874, 1.2419)
3	1.6000	0.2104	0.2104	(-0.0472, 0.4563)
4	1.4000	0.1253	0.1253	(0.0453, 0.3511)

Table 14. The four hand iterations at a glance. The pack best can wobble (iteration 2), but the best-so-far never worsens.

8. Key takeaways

- Three guides, one average. Every wolf steps toward the average of what alpha, beta, and delta suggest. No single leader dominates, which keeps the search stable.
- The parameter a runs the show. Starting near 2 it lets wolves roam widely (you saw W3 fling out to the boundary early); shrinking toward 0 it draws the pack tight around the leaders.
- The pack can wobble, the memory does not. Iteration 2 produced a worse pack best than iteration 1, yet the best-so-far held. Always keep the best point you have seen.
- Boundaries matter. A wolf pushed past an edge is clamped back, which keeps every candidate a legal solution.
- It converges. In four hand iterations the best f dropped from 29 to about 0.1253, landing next to the known optimum at (0, 0). The spreadsheet and code carry the same run further and reach f near 0.0231.